Resonance between Alfven Waves and Planetary Tides on the Sun

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Abstract. The tides on the Sun induced by the planets are generally considered to be too small to cause any measurable activity on the Sun. However, Seymour et al. (1992) proposed that the planetary tides might be amplified by resonating with the magnetic Alfven waves on the Sun. Their preliminary calculations showed that such resonances might exist. However, their simulations were carried out only in the photosphere and at the equator using a very simple solar magnetic field. We extend the magneto-tidal resonance theory to three dimensions by including the effects of latitude and depth and by using a more realistic solar magnetic field pattern. In particular, it is shown that, from solar minimum to maximum, the location of magneto-tidal resonance on the Sun moves toward the equator and toward the surface. The results reveal that planetary tides could be important in understanding the mechanism of solar activity and its periodicity.

Keywords: Sun, solar activity, planets, tide, Alfven wave

Introduction

Solar activity

Ever since it was discovered that the Sun is highly dynamic, scientists have been trying to understand what causes solar variability. Although the generation of a global solar magnetic field by the solar dynamo is not solved definitively, modification of the field by the differential rotation and meridional circulation is well established. According to the pioneering model put forward by Babcock (1961), the differential rotation forms toroidal field lines and the rise of such magnetic flux ropes appear in the photosphere as sunspots, regions of relatively colder and highly magnetized plasma, and as filaments in the corona. Using helioseismology, it was later found that this solar dynamo mainly operates near the tachocline at ~0.7 R_s where the rotational shear is maximum (Spiegel and Zahn, 1992). Advances in solar magnetohydrodynamics (MHD) suggested that the eruptive solar activity is the manifestation of the sudden release of large amounts of stored magnetic energy in twisted flux ropes via magnetic reconnection (Kopp and Pneuman, 1976). The migration of sunspots from mid-latitudes to lower latitudes during the course of a solar cycle is known as the butterfly diagram and was discovered by Maunder in 1904 (Babcock, 1961). This periodic latitudinal migration of sunspots is thought to be related to the meridional circulation pattern (Hathaway et al., 2003).

However, the root causes of various properties of the Sun revealed by observations (such as differential rotation, butterfly diagram, meridional circulation) as well as the actual mechanisms that cause both short-term solar activity (such as flares, coronal mass ejections, sunspots) and the long-term solar variability (such as the ~11 yr solar cycle) have no widely agreed-upon explanation. In particular, although buoyancy is thought to be a key factor (Parker, 1955), how exactly flux ropes intensify and emerge from the convection zone rising through the photosphere and into the corona and why these happen much more often at regular intervals (i.e., at solar maximum) still remain to be solved. Identification of what really causes solar activity is critical in understanding the Sun and improving space weather forecasting. Here, we investigate whether planetary tides on the Sun could actually become an important factor by getting into resonance with Alfven waves on the Sun.

Ocean tides

We first briefly describe the ocean tides on Earth before discussing the planetary tides on the Sun to illustrate how ocean tides, which are originally tiny, are amplified. On the earth, ocean tides are known as the rise and fall of sea levels caused by the gravitational attraction of the Moon and the Sun. Tidal force depends on the gravitational gradient, so it increases linearly with mass and falls off with the cube of distance (Takahashi, 1967). Tides on Earth manifest as two bulges in the oceans at opposite locations on Earth causing the semi-diurnal (~12 hr) ocean tide. The diurnal (~24 hr) ocean tide is a result of the tilt of Earth's rotation axis relative to that of the Moon's orbit and thus occurs only at mid and high latitudes.

On Earth, the tidal force of the Moon is about 10^{-7} times the Earth's gravitational force (and half of that for the solar tide) (Seymour et al., 1992); yet, it still causes considerable tidal waves in the ocean, especially due to the shallow water and funneling effects at shores and due to resonance at specific locations. Each body of water has a natural oscillation period determined by its shape and size. If a body of water has a natural oscillation period of ~12 hr (or ~24 hr), it resonates with the semi-diurnal (diurnal) tide, and the tidal heights are greatly amplified. For example, although the maximum theoretical tidal height due to the combined effects of Moon and Sun is only about 0.8 m, the tides at Bay of Fundy near Nova Scotia can reach heights as much as 16 m due to its shape and ~12 hr natural oscillation period that causes resonance with the semi-diurnal lunar tide (Garrett, 1972).

Planetary tides on the Sun

The vertical and horizontal tidal forces on the Sun caused by the planets are given by the following equations (from Takahashi, 1967):

$$Fr = GmMR_s(3\cos^2\theta-1)/D^3 \text{ (vertical tidal force)} \quad (1)$$

$$Ft = -1.5GmMR_s\sin2\theta/D^3 \text{ (horizontal tidal force)} \quad (2)$$

G (gravitational constant)
m (mass of solar particles)
M (mass of the planet)
R_s (solar radius)
D (distance between the planet and the Sun)
θ (latitude on the Sun)

The relative magnitudes of the planetary tides on the Sun are given in Table 1. It can be seen that the theoretical magnitude of the tides on the Sun caused by Mercury, Venus, Earth, and Jupiter are all comparable and much larger than the tides caused by the rest of the planets.

TABLE 1. Magnitudes of the planetary tides on the Sun relative to the Earth

	Period (yr)	Mass (M_E)	Distance (AU)	Tidal Force on Sun
Mercury	0.24	0.06	0.31-0.47	**0.55-1.85**
Venus	0.62	0.82	0.72	**2.18**
Earth	1	1	1	**1**
Mars	1.88	0.11	1.52	0.03
Jupiter	11.86	318	5.20	**2.26**
Saturn	29.46	95	9.54	0.11
Uranus	84.01	15	19.18	0.002
Neptune	164.8	17	30.06	0.0006

Planetary tidal gravities on the Sun are about 10^{-12} of its surface gravity which is much smaller than the lunar tide on Earth which is 10^{-7} of Earth's gravity (Grandpierre, 1996). However, there are various factors that can amplify the planetary tidal effects on the Sun:

- Tides from two or more planets add up during planetary alignments (i.e. conjunctions or oppositions).
- The duration of tides is much longer due to slow solar rotation. Unlike the oceans on the Earth, the solar atmosphere consists of plasma for which small accelerations for extended periods can build up and have large consequences (e.g. sudden release of stored magnetic energy via reconnection).
- Tides are greatly amplified in the corona due to increase in tidal force and decrease in solar gravitational force. For example at a height of 2 solar radii, the tidal force is 2 times larger and solar gravitational force is 4 times smaller than solar surface, so tidal effect becomes 8 times larger.
- Horizontal tidal forces can move large volumes of solar plasma along the field lines without much resisting force. Converging field lines can concentrate plasma "tidal waves" into a narrow region, thereby increasing tidal heights considerably due to funneling effect.
- Similar to the ocean tides in Bay of Fundy, a possible resonance between the natural oscillations on the Sun and planetary tides could greatly amplify the tidal effects, which is the topic of this study.

Planets and solar activity

Since the discovery of solar cycle in 1843 by Schwabe, the similarity of its ~11 yr period to the 11.86 yr orbital period of Jupiter has motivated researchers to find a link between planetary motion and solar activity. After all, it has not been easy to explain the periodic properties of the Sun that persist for centuries by turbulence and internal dynamics only. The periodicity and long-term stability of planetary orbits could provide the missing link.

During the last century, various researchers studied the relation between both short-term and long-term solar activity and planetary orbital motions and observed correlations between the two. This correlation was usually explained by two mechanisms:

- The Sun's irregular motion around the solar system barycenter due to the mass displacement caused by the giant planets (Jupiter, Saturn, Uranus, Neptune). The Sun's orbital angular momentum changes considerably during its motion. It is suggested that this could affect the solar rotation via spin-orbit coupling which could be the cause of the long-term periodic solar activity (~11 yr and longer cycles).
- The tides on the Sun caused by the planets (Mercury, Venus, Earth, Jupiter). It is suggested by many that the tides could trigger short-term solar activity such as sunspots, flares, and CMEs. Some researchers also found an ~11 yr cycle in planetary tides.

Majority of the studies on planets and solar activity focused on the relation between solar motion around barycenter and solar cycle. Others studied the effect of planetary positions on sunspots and flares (e.g., Bigg, 1967; Blizard, 1969; Ambroz, 1971; Seymour et al., 1992; Grandpierre, 1996; Hung, 2007). Most of these studies presented empirical evidence for a relation between solar activity and planetary configurations, either suggesting planetary tides as the link or without providing any physical explanation. A common explanation of how planets can cause sunspots or flares is that the planetary tides on the Sun could cause the magnetic flux ropes on the Sun to rise and eventually erupt. After all, the Babcock model (Babcock, 1961) can explain how the solar magnetic field is twisted and amplified due to differential rotation; and the planetary tides can pull these flux ropes triggering the release of previously stored magnetic energy via reconnection. However, planetary tides are too tiny for pulling the flux ropes against solar gravity, so an amplification mechanism is needed. One of the most convincing physical theories was developed by Seymour et al. (1992) who suggested a possible tidal amplification mechanism which is introduced next.

Magneto-Tidal Resonance Theory

Seymour et al. (1992) put forward a theory based on resonant amplification of Alfven waves on the Sun by planetary tides. It was proposed that the solar activity can be caused by resonance between the planetary tides and solar magnetic Alfven waves when their angular speeds are nearly equal, therefore greatly amplifying the tides. For simplicity, they assumed that the solar magnetic field is parallel to equator (i.e., toroidal) as a result of differential rotation and calculated the equatorial Alfven wave speeds at the top and bottom of the convection zone that satisfy the resonance condition for planetary tides. They showed that the magnetic field strengths required for sustaining such Alfven wave speeds (~700 G in the photosphere and ~7000 G at the tachocline) are similar to the actual observed values (Seymour et al., 1992). They then varied the solar magnetic field (and thus Alfven wave speed) sinusoidally with a ~11 yr period and showed that, during the course of a solar cycle, solar Alfven waves on a specific "magnetic canal" (a toroidal field line) at the equator sequentially get into resonance with the tides caused by each planet due to the different orbital angular speed of each planet.

According to the calculations by Seymour et al. (1992), the tidal height (u) at a location s(θ,φ) on a magnetic canal parallel to the solar equator at solar surface is given by:

$$u = V_A^2 H\cos^2\theta\cos 2(wt+\varphi+e) \,/\, 2(V_A^2 - w^2 R_s^2 \cos^2 \quad (3)$$

$$V_A^2 = B^2/\mu_o\rho \text{ (Alfven wave speed)} \quad (4)$$

$w = w_s - w_p$ (angular speed of planet relative to Sun at s)

B (solar magnetic field strength at s)

ρ (mass density of charged particles at s)

$H = G_t/g$ (ratio of tidal to gravitational acceleration)

$G_t = 2GMr/D^3$ (tidal acceleration at equator)

g (solar gravitational acceleration at surface)

R_s (solar radius)

θ (latitude of magnetic canal)

φ (longitude of s)

t (time)

e phase factor;e = 0 if the planet is facing s at t=0)

On such a magnetic canal, resonance occurs if the Alfven wave speed, V_A, is nearly equal to $wR_s\cos\theta$, in which case, planetary tides are greatly amplified. As the solar cycle progresses the solar magnetic field gradually increases mainly due to the winding of the field lines by differential rotation as originally suggested by Babcock (1961) and also by the converging and twisting of flux ropes. As the solar cycle progresses, Alfven wave speed changes; and, at a specific latitude, each planetary tide (from the fastest orbiting planet to the slowest) gets into resonance with the Alfven wave in sequence. Seymour et al. (1992) showed that an Alfven wave propagating with a speed of ~2 km/s around the equator in the photosphere does get into resonance with the tidal planets and that even a 5% change in Alfven wave speed can greatly affect the resonance condition at a particular location on the Sun.

Method

The magneto-tidal resonance theory put forward by Seymour et al. (1992) revealed that gravitational tides caused by the planets might actually be important in the generation mechanism of sunspots, CMEs, and flares. However, the simulations they did were too simplified as explained below:

- They used a simple one-dimensional solar magnetic field that is parallel to the equator and that varies sinusoidally with time.
- The effect of differential rotation on angular speeds and on magnetic field orientation was ignored.
- The calculations were made only at a specific point (i.e., sub-planetary point on solar surface).
- The effect of each planet was calculated for an ideal circular planetary orbit parallel to solar equator.

The Alfven wave speed (Equation 4) depends on plasma mass density and magnetic field which depends not only on the location on the Sun (e.g., latitude and height/depth) but also varies with time (due to solar cycle). Moreover, the resonance depends on the "angular" speed of the Alfven wave (which depends not only on magnetic field intensity but also on field geometry and location on the Sun) relative to the angular speed of planetary tide (which depends on the particular planet and its orbital position due to eccentricity). If the latitude and depth factors are included, then it can be shown that tides induced by different planets can simultaneously resonate with the solar Alfven waves but at different latitudes and depths which shift gradually as the global solar magnetic field intensity and geometry changes during the course of a solar cycle. Accordingly, we improve the resonant tide theory (given in Equation 3) by using a more realistic solar magnetic field pattern and variation and extend it to three dimensions by including the effects of latitude and depth as well as differential solar rotation. Next, using a simple model based on equations given by Babcock (1961), we demonstrate that the location (e.g., depth and latitude) of resonance between the planetary tides and solar Alfven waves depend on the planet and vary with time.

Latitude dependence

The latitudinal dependence of resonance can arise from five factors:

- Differential solar rotation (e.g., solar convection zone rotates at a slower rate at higher latitudes).

- Solar magnetic field intensity (and thus Alfven wave speed) varies with latitude.
- An Alfven wave with a specific speed parallel to the equator has a higher angular speed at higher latitudes due to the shorter distance traveled.
- The parallel angular speed of Alfven waves depend on the tilt of the magnetic field which varies with latitude. Solar magnetic field (after a few rotations) becomes nearly parallel with equator at mid- latitudes in contrast to the field at very low (equatorial) and very high (polar) latitudes which always remain nearly meridional.
- Vertical and horizontal planetary tides are largest near the equator and mid-latitudes, respectively. At high latitudes, horizontal tides are negligible and vertical tides become negative.

Furthermore, the resonant latitude varies in time due to the following:

- Variation of solar magnetic field intensity (and thus Alfven wave speed) with time (increases due to differential rotation, convergence, and twist and decreases due to field cancellation and relaxation).
- Variation of solar magnetic field pattern (which gradually changes from a dipole to a toroidal configuration due to differential rotation). As the toroidal component becomes larger, the angular speed of the Alfven wave parallel to the solar equator increases.
- The resonance latitude would slightly change in time due to elliptical orbits of the planets, but for simplicity, here we assume circular planetary orbits with constant angular speeds.

To define these relations, we start with the following equations taken from Babcock's model (Babcock, 1961):

$$w_S = 14.4° - 2.8° \sin^2\theta \quad (5)$$

(solar differential rotation in degrees per day)

$$\tan(\gamma) = 17.6.t.\sin(2\theta) \quad (6)$$

(γ is the angle of B with meridian)
(t is time in years since solar min)

$$B = B_o / (\cos\gamma\cos\theta) \quad \text{for } -30°<\theta<30° \quad (7)$$

(θ is the solar latitude)

However, representation of solar horizontal magnetic field with Equation 7 is only valid for low latitudes since it goes to infinity for high latitudes which is not realistic. For higher latitudes, we use $B=B_o(1+k\sin\theta)/\cos\gamma$ which is more reasonable. The transformation of poloidal field into toroidal form is represented by Equation 6. The field is mostly parallel to the equator (i.e., γ~90°) except for very early in solar cycle (t~0) (for which the field is poloidal for all latitudes) and at very low and very high latitudes (θ~0° and θ~90°) (for which the field always remains poloidal). We then calculate the dependence of Alfven wave angular speed on latitude and time assuming that density (ρ) in Equation 4 does not change with latitude. We also assume longitudinal symmetry. For resonance, instead of the actual Alfven wave speed (V_A), the angular speed of Alfven waves parallel to equator, which we define as w_A, is important. We calculate it as follows:

$$w_A = V_{A//}/R_s = V_A \sin\gamma/R_s$$
$$\sim B(\sin\gamma) \sim (1/\cos\gamma\cos\theta)(\sin\gamma) \sim \tan\gamma/\cos\theta$$
$$\sim t\sin(2\theta)/\cos\theta \sim t\sin(\theta) \text{ for } \theta<30° \quad (8)$$

For $\theta>30°$, we use $w_A \sim B(\sin\gamma) \sim (1+k\sin\theta)\tan\gamma \sim t(1+k\sin\theta)\sin(2\theta)$

$$\text{Resonance condition: } w_S(\theta)-w_A(t,\theta) = w_P \quad (9)$$

(w_P is the angular speed of the planet)

Depth dependence

The resonance between Alfven waves and planetary tides depend on depth due to two main reasons:

- The mass density of charged particles and magnetic field intensity (ρ and B in Equation 4) increases with depth. As a result, the Alfven wave speed changes with depth.
- The radius of the circular magnetic canal decreases with depth (as well as with latitude), so the angular speed of the Alfven wave increases.

For simplicity, we assume that the magnetic field geometry does not change much with depth which is mostly valid in the convection zone where the differential rotation still applies. However, the field geometry changes considerably at the tachocline where the radial rotational shear is maximum. So our calculations are valid only above the tachocline where the angular speed of Alfven waves is not affected by the radial tilt of field lines. We also assume that the density and magnetic field both increase exponentially with depth in the convection zone. The R_s in Equation 8 is replaced by R and the B and ρ in Equation 4 are replaced by:

$$\rho(R)= \rho_o e^{k(1-R/R_s)} \quad B(R)=B_o e^{k(1-R/R_s)} \quad (10)$$

R (radius of the shell, $0.7R_s$ at tachocline)
B_o (magnetic field strength at the surface)
ρ_o (mass density of charged particles at the surface)
k (coefficient to adjust the boundary values)

It is seen from Equation 8 that these changes introduce the following factor to w_A.

$$w_A(R) = w_A e^{0.5k(1-R/R_s)}/(R/R_s) \quad (11)$$

From this equation, it is clear that the angular speed of the Alfven wave increases considerably with depth (i.e., as R decreases), thereby affecting the latitude of resonance with planetary tides.

Results

For a magnetic canal parallel to the equator, the resonance condition in Equation 9 results in the following:

$$S = B/\rho^{0.5} \quad (12)$$
$$B = B(d, \theta, t)$$
$$\rho = \rho(d)$$

$d=R/R_s$ (d=0.7 at tachocline, and d=1.0 at the surface)

$$V_A = S(\mu_o^{-0.5}) = (892)S \quad (13)$$

$$w_A = V_A/R = V_A/dR_s = (892)S/d(7x10^8) \text{ (rad/s)} \quad (14)$$

$$w_A(^0/day)=w_A(rad/s)\ (180°/\pi)\ (86400\ s/day) \quad (15)$$

$$w_A=(S/d)(892x180x86400)/(7x10^8 x\pi)=6.3(S/d) \quad (16)$$

$$w_A= 6.3(S/d) = (w_S-w_P) \text{ (resonance condition)} \quad (17)$$

From Equation 5, the solar angular speed, wS, varies from ~14.4 °/day at the equator to ~11.6 °/day at the poles. The angular speed of the tidal planets are in the range of 0-5 °/day (4.11° for Mercury, 1.59° for Venus, 0.98° for Earth, 0.08° for Jupiter). From this equation, it is possible to get an idea of required values of magnetic field (B) and density (ρ). For example, at the equator (θ=0°) and surface (d=1.0), for Alfven waves to resonate with the planetary tides caused by Earth the ratio S=B/ρ0.5 needs to be equal to (wS-wP)(d/6.3)=(14.4-0.98)/6.3=2.13. So, for instance, a typical sunspot magnetic field of 0.3 T and surface density of 0.02 kg/m3 would satisfy the resonance condition. The Alfven waves in such magnetic canal would have a speed of 2.13×892=1.9 km/s (an angular speed of 13.4°). The rotational speed of the Sun at the equator is ~2 km/sec (or 14.4 °/day). And at the solar equator the speed of the tide caused by Earth is ~0.1 km/s (or ~1 °/day). So, with these values, resonance occurs (i.e., wS-wA=wP) and as a result, the tides caused by Earth would be amplified. Near the tachocline below the equator (d=0.7, θ=0°) B and ρ increase a lot and the ratio S decreases to 2.13x0.7=1.5. So, for example, a typical tachocline B value of 3 T and density value of ~4 kg/m3 would satisfy the resonance condition for the tides caused by Earth near the tachocline. Note that, when B increased 10 times, ρ had to increase 200 times to have a constant Alfven wave speed (i.e., to keep resonance) which is a reasonable depth variation of B and ρ (e.g., Figures 16 and 18 of Fan, 2009).

Latitude dependence

The simulation result for the latitudinal dependence of angular speed of the Alfven wave and solar rotation is shown in Figure 1 for the surface (i.e., photosphere) at solar minimum. For this calculation, the values used for the magnetic field at the equator (B_o) and the density (ρ_o) are 0.1 T and 0.03 kg/m^3, respectively.

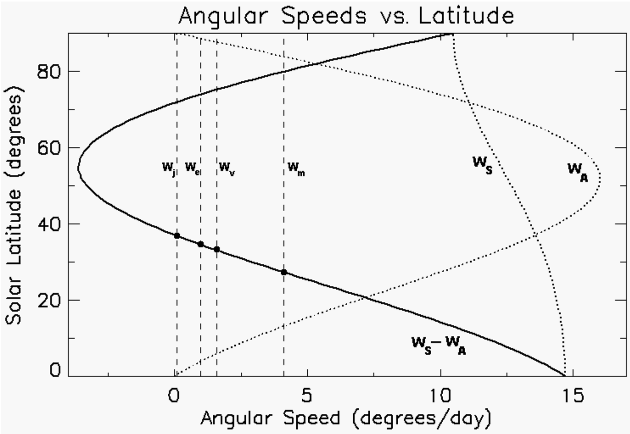

Fig.1. Latitudinal dependence of the angular speeds of Alfven waves, the Sun, and tidal planets in the photosphere at solar minimum for B_o=0.1 T and ρ=0.03 kg/m^3. The w_S-w_A curve and its intersections with the planetary angular speeds (i.e., resonance locations) are also shown

The latitude dependences of solar rotational and Alfven wave angular speeds are shown with wS and wA, respectively. The angular speeds of the tidal planets (Mercury, Venus, Earth, and Jupiter) do not change with latitude so they appear as vertical lines. For each planet, the latitude of tidal resonance is given by the intersection of the corresponding vertical line with the curve wS-wA which are shown by black dots. Although each planet has two intersection points, one of them is at a very high latitude and can be ignored as the tides at such high latitudes are negligible. As for the other intersection point, it can be seen that tides due to faster (slower) planets gets into resonance with solar Alfven waves at lower (higher) latitudes. It should be noted that the calculations were made at a fixed depth (at surface) and density was assumed not to vary with latitude.

The variation of wS-wA in time (due to the variation of wA) is shown in Figure 2. For simplicity, the orbits of planets are assumed circular, so the angular speeds of planetary tides are taken as constant. Also shown in the figure is the decrease of the planetary tidal force with latitude (labeled as "t") due to the ~cos2θ factor in Equation 3. The result is shown only up to 40° latitude as the resonances at high latitudes are not important as explained earlier.

According to Equations 6 and 7, as the time increases, the magnetic field increases and becomes more toroidal both of which result in faster angular speed of Alfven waves. As a result, the wS-wA curve shifts towards left. It can be seen in Figure 2 that during the course of a solar cycle, the resonance location for each planet gradually moves to lower latitudes as a result of increased Alfven wave angular speed.

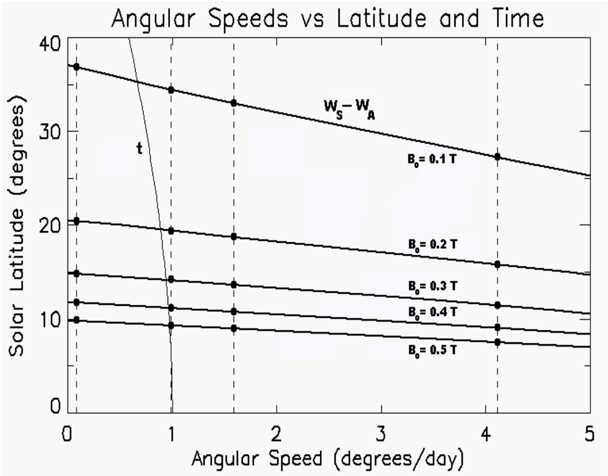

Fig.2. The dependence of the tidal resonance latitude on the planet and the angular speed of the Alfven wave which varies with time due to changes in magnetic field intensity and configuration. Resonance locations are shown with black dots. Each wS-wA curve is calculated using a different solar equatorial magnetic field. It is assumed that solar magnetic field intensifies in time and becomes more toroidal. Therefore the angular speed of Alfven waves (wA) increases and the relative angular speed (wS-wA) decreases. This causes the resonance locations to gradually move to lower latitudes. Also shown in the plot is the decrease of the planetary tidal force with latitude (t).

Figure 3 shows the resonance latitude variation with time for tidal planets corresponding to the intersection points in Figure 2 which looks similar to a typical butterfly diagram. We note that, in our model (e.g., Equations 6 and 7), the solar magnetic field gradually becomes more toroidal and intensifies and does not return to the dipole (e.g., via field cancellation). As a result, the resonance latitude keeps decreasing

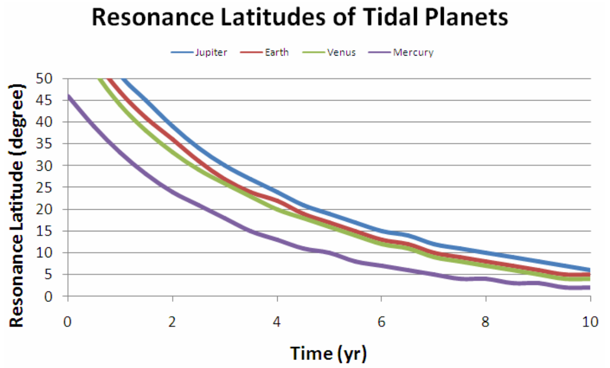

Fig.3. The variation of resonance latitude with time for tidal planets. The minor wiggles are artifacts due to the low latitude resolution (1°) used in the calculations. Resonance locations are shown with black dots.

Depth dependence

As mentioned earlier, the Alfven waves propagate faster with increased depth. Consequently, similar to the shift in time in Figure 2, the wS-wA curve shifts toward lower values of angular speed and intersects the vertical planetary lines at lower latitudes with increasing depth. So, the latitude of resonance between the Alfven waves and planetary tides decreases with depth. This dependence of resonance latitude on depth is illustrated in Figure 4 for a magnetic field of 3 T and density of 1 kg/m^3 near the tachocline below the equator at solar minimum. Both the density and magnetic field are assumed to decrease exponentially with height using Equation 10. As shown with an arrow, as the time (and thus B) increases, the curves shift toward right to increased heights or lower depths for specific latitude and to lower latitudes for a specific depth.

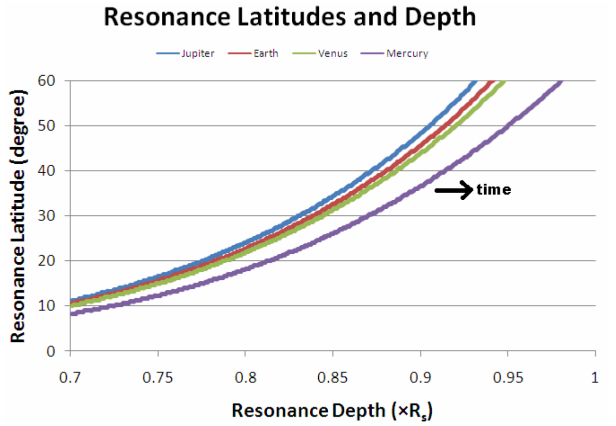

Fig.4. The variation of resonance latitude with depth and time for tidal planets for a magnetic field of 3 T and density of 1 kg/m^3 at the tachocline below the equator at solar minimum

Discussion

Latitude dependence

Three important features of the observed sunspot distribution on the Sun can be explained with the magneto-tidal resonance theory:

- Sunspots gradually move to lower latitudes during a solar cycle (the butterfly diagram). This can be explained by the gradual shift of the magneto-tidal resonance location to lower latitudes.
- Sunspots hardly appear above 40° latitude. This can be due to a couple of reasons.
  - The tidal forces become too small at high latitudes.
  - The minimum Alfven wave speed (e.g. at solar min in the photosphere) is not small enough to have resonances above 40°. In other words, the Alfven wave angular speed increases with latitude and above a certain latitude might become larger than the angular speed of the Sun and thus rotate in the opposite direction of the planets.
- Sunspots almost never appear near solar equator. Since the solar magnetic field at the equator (and poles) does not have a component parallel to the equator (i.e., γ=0° for θ=0° and 90° in Equation 6), at the equator the angular speed of Alfven wave parallel to equator is zero. As a result, at the equator, wS-wA= wS which is ~3 times faster than the fastest planet. So, regardless of how fast the Alfven waves are they can not get into resonance

with planetary tides near the equator (or the poles) due to the geometry of the magnetic field.

It is observed that very early in a solar cycle the field lines are mostly poloidal so no resonances can occur at any latitude; however as the field becomes more intense and more toroidal, resonances due to planets start to occur at high-latitudes and gradually propagate to lower latitudes. The secondary resonances at high latitudes are not important as the tides become too small at high latitudes. The resonances never occur near the equator as the field there is always poloidal. The resonances with faster planets always occur at lower latitudes than with slower planets. It should be mentioned that this preliminary simple model assumes the solar magnetic field keeps winding and intensifying during the course of a solar cycle. To be more accurate, after reaching solar maximum, the solar magnetic field should start weakening and gradually return to a poloidal configuration. However this complex dynamic process involving field cancellations and relaxations via reconnection, merging, and migration of active regions (Leighton 1969) is beyond the scope of this study.

Depth dependence

The results on the depth variation of resonance latitude show that, deep in the convection zone, resonances occur at lower latitudes closer the equator. Assuming density and magnetic field both decrease exponentially with height, the resonance location rapidly moves to higher latitudes higher in the convection zone. This might be another explanation of why sunspots are hardly observed near the equator at the surface. If the resonant flux ropes rise vertically (i.e., constant latitude), they will get out of resonance quickly at least until they get into resonance with another, faster moving planet closer to the surface. In addition, as B increases with time (i.e., from solar min to solar max), the resonance locations move toward the surface for specific latitude and to lower latitudes for specific depth.

On the other hand, the dynamo theory suggests that the sunspots are created at the bottom of the convection zone (i.e., tachocline) where the radial rotational shear is maximum. According to the most common explanation, the toroidal flux ropes created at the tachocline twist, intensify, and buoyantly rise in the convection zone and appear as sunspots when they reach photosphere (Babcock, 1961).

The result for a flux rope will be different as the density variation with depth (instead of magnetic field intensity) would be the dominant factor. According to our model, such flux ropes at the tachocline can resonate with tides and rise. While they are rising, the magnetic field decreases little compared to the density which decreases exponentially. As a result, the Alfven waves become faster (in contrast to the case with exponentially decreasing B). So, to keep the resonance, the flux ropes have to migrate toward the equator where the Alfven wave speed is lower. So, the prediction of this model on the depth variation of resonant latitude depends mainly on how rapidly the magnetic field decreases with height. It is likely that the density decreases much more rapidly with height than the magnetic field, in which case the result shown in Figure 4 would not be valid as the resonance latitude might stay nearly constant or even decrease with height depending on how fast the density is decreasing compared to the magnetic field. According to the numerical model result shown in Figure 18 of Fan (2009), the Alfven wave speed in a flux rope remains constant from the tachocline at $0.7R_s$ up to $\sim 0.9R_s$ and then decreases toward the surface. This means that the ratio $B/\rho^{0.5}$ remains constant through most of the convection zone suggesting that the increase in B with depth is indeed much slower than the increase in ρ.

Conclusions

These results show that the planetary tides do often get into resonance with the magnetic Alfven waves on different parts of the Sun. The magneto-tidal resonance theory can be useful in understanding various properties of the Sun (such as the butterfly diagram) and suggests a physical mechanism for sunspot and active region formation. In these simulations, tilts and eccentricities of planetary orbits were ignored. Since the resonances are very sensitive to small changes in parameters, it is necessary to include the slight variability of planetary angular speeds in the model. Furthermore, the orbital tilts of the planets might be useful in explaining the north-south asymmetry of solar activity. Most importantly, the periodicities detected in solar activity (including the ~11 yr solar cycle) could be explained by periodic enhancements of tides due to periodic alignments of planets and due to the periodic changes in the orbital angular speeds of planets due to their elliptical orbits. Using ephemerides data for the planetary orbits and actual data for the Sun (density, magnetic field orientation and intensity) would allow the model to make predictions of actual resonance locations. Once this is accomplished, it would be possible to compare the predictions based on the model with actual solar activity to test whether the magneto-tidal resonance theory is important. If this theory is found to have merit, the predictability of planetary orbits would then enable precise and long-range forecasting of solar activity.

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