

# The study of time series of monthly averaged values of F10.7 from 1950 to 2010

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**Abstract.** Prior to 1947, the activity of the Sun was assessed by the relative numbers of sunspots ( $W$ ). The 10.7 cm radio emission (frequency of 2.8 GHz) for observations of the variability of radiation of chromosphere and the lower corona (F10.7) became used from 1947. For the F10.7 are available more detailed observational archive data, so this activity index more often than the other indices is used in the prediction and monitoring of the solar activity. We have made the analysis of time series of F10.7 with the use of different mother wavelets: Daubechies 10, Symlet 8, Meyer, Gauss 8 and Morlet. Wavelet spectrum allows us not only to identify cycles, but analyze their change in time. Each wavelet has its own characteristic features, so sometimes with the help of different wavelets it can be better identify and highlight the different properties of the analyzed signal. We intended to choose the mother wavelet, which is more fully gives information about the analyzed index  $F_{10.7}$ . We have received, that all these wavelets show similar values to the maximums of the cyclic activity. However, we can see the difference when using different wavelets. There are also a number of periods, which, perhaps, are the harmonics of main period. The mean value of 11-year cycle is about 10.2 years. All the above examples show that the best results we get when using wavelets Morlet, Gauss (real-valued) and multiparameter family of wavelets Morlet and Gauss (complex-valued).

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**Keywords:** Solar cycle: observations, solar activity indices, wavelet spectrum;

## Introduction

It has become of great practical and societal importance to predict solar activity and space climate. There are some important global indices of solar activity which allow us to monitor the situation on the Sun and to build various forecasts. We have studied these indices and their mutual correlation during the solar cycles 21 -23 in Bruevich and Yakunina (2011). It is well known that there is a strong correlation of 10.7 cm radio flux with the other global indices of solar activity. This is due to the fact that the sources of 10.7 cm radio flux are approximately at the same altitudes in the solar atmosphere and are characterized by the same parameters of the surrounding plasma. So 10.7 cm radio flux is a good measure of the general solar activity. There is strong correlation also between 10.7 cm flux and full-disc X-ray flux. The correlation is best when the activity is high, but when the activity is low, the X-rays are too weak to be detected, while some 10.7 cm emission in excess of the so called Quiet Sun Level is always present (Kruger 1979). In Bruevich and Yakunina (2011) we also confirmed these conclusions.

## 10.7 - cm solar radio flux activity index

The solar  $F_{10.7}$  observations started in 1947 in Ottawa, Canada and are maintained to this day. At present this important global index is measured at the Dominion Radio Astrophysical observatory. The 10.7 - cm radio flux is a good indicator of the solar chromospheric, transition region, and coronal EUV fluxes.  $F_{10.7}$  changes depend on the bright solar active regions whose energies are absorbed in the earth's thermosphere and have a great influence on the climate on Earth. We used in our paper the data published in Solar-Geophysical Data Reports (2009)

and National Geophysical Data Center, Solar Data Service (2013).

The 10.7 - cm emission from the whole solar disc can be separated on the basis of characteristic time-scales into 3 components: (i) transient events associated with activity having duration less than an hour; (ii) slow variations in intensity over hours to years, so-called S-component; (iii) a minimum level below which the intensity never falls (the Quiet Sun Level), see Tapping and De Tracey (1990). Donnelly et al. (1983) confirmed the strong correlation of 10.7 cm S-component with full-disc flux in Ca II and Mg II.

With the help of 10.7 cm radio flux we can successfully predict the integrated flows in the UV and EUV, see (Chapman and Neupert, 1974; Donnelly et al., 1983; Nicolet and Bossy, 1985; Lean, 1990).

Two different sources of 10.7 cm radio flux are: thermal bremsstrahlung (free-free radiation) and gyro-radiation (due to electrons radiating when changing direction by gyrating around magnetic fields lines).

According to Gaizauskas and Tapping (1998), the minimum level component (iii) can be seen when the number of sunspots is equal to zero and the local magnetic fields are negligible. This component is defined by the free-free source. In case when the local magnetic fields become strong enough at the beginning of the rise phase of the solar cycle and sunspots appear, the gyro-radiation source of  $F_{10.7}$  radio flux begins to prevail over the free-free source so (i) and (ii) components begin to grow strongly. The S-component comprises the integrated emission from all sources on the solar disc. It contains contributions from free-free and gyroresonance processes, and perhaps some non-thermal emission.

The history of wavelets is not very old, at most 15 to 20 years. There are lots of successes for the community to share. Fourier techniques were stimulated by the appearance of windowed Fourier methods that operate locally on a time-frequency approach. The wavelets bring their own strong benefits to that environment: a local outlook, a multiscaled outlook, interaction between scales, and a time-scale analysis. They demonstrate that sines and cosines are not the only useful functions and that other bases made of weird functions serve to look at new signals, as strange as most fractals or some transient signals.

We tried to choose the wavelet most useful for the analysis of observational data of different indices of solar activity. In this paper we analyze the different mother wavelets for the study of F<sub>10.7</sub> data (as a measure of the general solar activity).

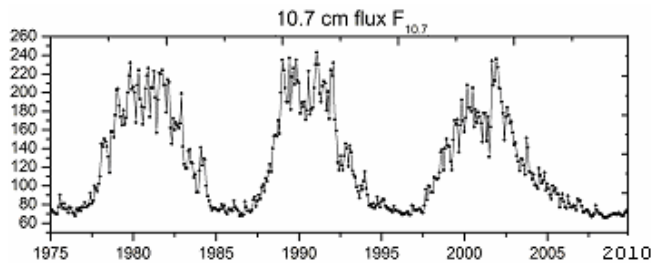


Figure 1: The time series of monthly average of F<sub>10.7</sub> from 1975 to 2010. According to National Geophysical Data Center of Solar and Terrestrial Physics data.

Modern methods of spectral analysis, in particular wavelet analysis, allow us to successfully carry out the processing of data of observations of the solar activity on different time scales Morozova et al. (1999).

In this paper we make an analysis of the time series of F<sub>10.7</sub> with the use of different mother wavelets: Daubechies 10, Simlet 8, Meyer, Gauss 8 and Morlet (real and complex). It's known that Fourier analysis consists of breaking up a signal into sine waves of various frequencies. Similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or mother) wavelet.

The wavelets are the localized functions constructed with the help of one so-called mother wavelet  $\psi(t)$  by shift operation on an argument (b) :

$$\Psi_{ab}(t) = (1/\sqrt{|a|}) \cdot \psi((t-b)/a)$$

and a scale change (a):  $\psi((t-b)/a)$

The wavelet time-scale spectrum is a two-arguments function C(a,b) where 'a' is measured in reversed-frequency units, and 'b' is measured in time units:

$$C(a,b) = (1/\sqrt{|a|}) \int_{-\infty}^{\infty} s(t) \cdot \psi((t-b)/a) dt$$

### The choice of mother wavelet for our F<sub>10.7</sub> study

Wavelet analysis is successfully applied for the processing of time series of astronomical observations. In Vityazev (2001) the possibility of using various mother

wavelets for a set of astronomical applications was analyzed. The choice of wavelet is dictated by the signal or image characteristics and the nature of the application. If you understand the properties of the analysis and synthesis wavelet, you can choose a wavelet that is optimized for your application. Wavelet families vary in terms of several important properties. Examples include:

- ✓ support of the wavelet in time and frequency and rate of decay;
- ✓ symmetry or antisymmetry of the wavelet. The accompanying perfect reconstruction filters have linear phase;
- ✓ number of vanishing moments. Wavelets with increasing numbers of vanishing moments result in sparse representations for a large class of signals and images;
- ✓ regularity of the wavelet. Smoother wavelets provide sharper frequency resolution. Additionally, iterative algorithms for wavelet construction converge faster.

### Morlet wavelet

The Morlet wavelet is suitable for continuous analysis. The Morlet wavelet is a plane wave, modulated by Gaussian function:

$$\Psi(t) = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{t^2}{a^2}} e^{i 2 \pi t}$$

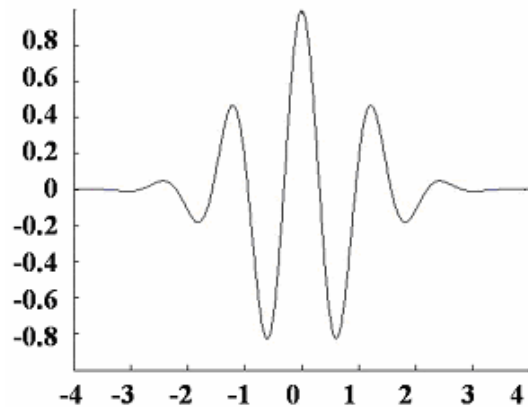


Figure 2a: The Morlet wavelet function  $\Psi$ .

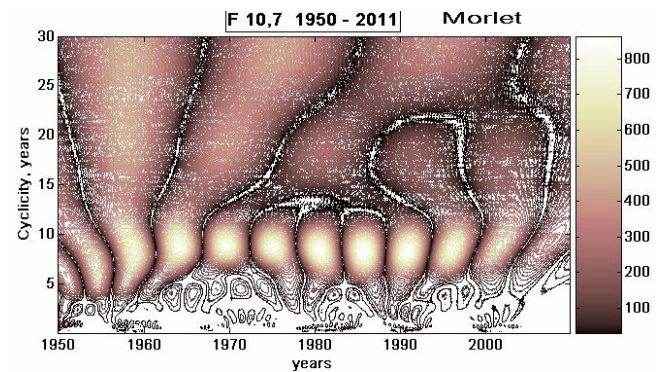


Figure 2b: The analysis of time series of monthly averaged F<sub>10.7</sub> with the use of Morlet mother wavelet.

In Figure 2a we show the Morlet wavelet function. Figure 2b demonstrates the results of wavelet analysis of the time series of monthly averaged F10.7. Plane XY corresponds to the time-frequency plane (a, b): a - Y (Cyclicality, years), b - X (Time, years). The  $C(a,b)$  coefficients characterizing the probability amplitude of regular cyclic component localization exactly at the point (a, b), are laid along the Z axis. In Figure 2b we see the projection of  $C(a,b)$  to (a, b) or (X, Y) plane. This projection on the plane (a, b) with isolines allows to trace the changes of the coefficients on various time scales and reveals a picture of local extrema of these surfaces. It is the so-called skeleton of the structure of the analyzed process. In Vityazev (2001) preference is given to the Morlet mother wavelet for processing of time series of astronomical observations. The interpretation of Morlet-wavelet images is similar to the interpretation of the results of Fourier analysis of data sets. We can also note that the configuration of Morlet wavelet is very compact in frequency, which allows us the biggest accurately (compared with other wavelets) determining the localization of the instantaneous frequency of the observed signal. We can see the main 11-yr cycle of activity. The most probable value of this cyclicality is about 10 years. We can also see a set of quasi-biennial cycles inside every 11-yr cycle which have durations that vary from 3 to 2.5 year.

**Daubechies wavelet**

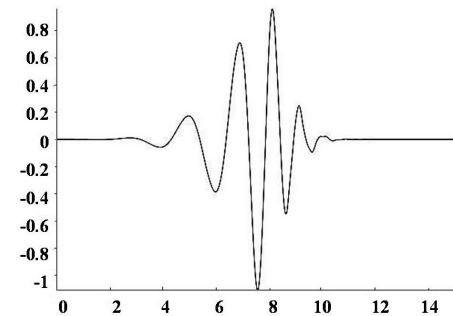


Figure 3a: The Daubechies 10 wavelet function  $\Psi$ .

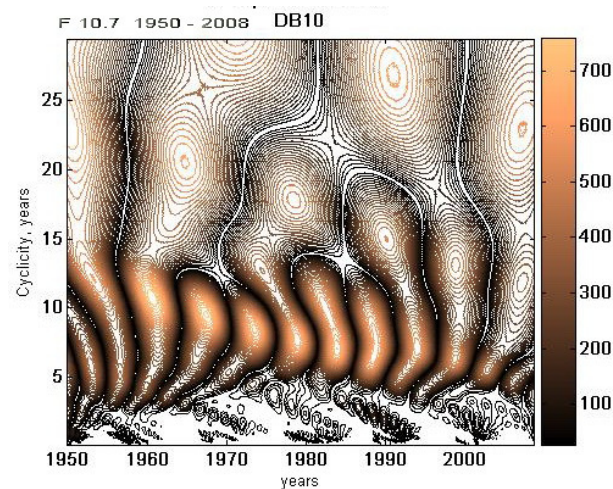


Figure 3b: The analysis of time series of  $F_{10.7}$  with the use of Daubechies 10 mother wavelet.

In Figure 3a we can see the Daubechies 10 wavelet function. In Figure 3b we demonstrate the analysis of the radio emission  $F_{10.7}$  with the help of Daubechies 10 mother wavelet. This study of the time series of  $F_{10.7}$  shows that the previous cycles affect the subsequent cycles. This is connected with the peculiarity of this wavelet, its wider coverage of the sample observations studied. But such a wide filter leads to more blurred values which determine the maximum probability of determination of the duration of the cycle.

**Simlet wavelet**

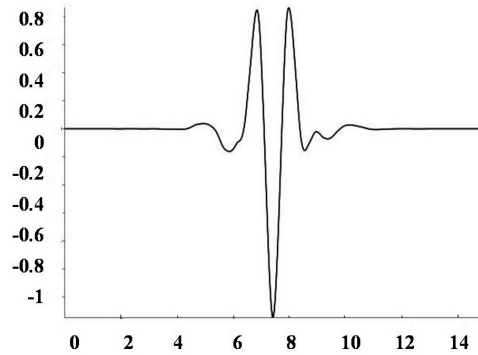


Figure 4a: The Simlet 8 wavelet function  $\Psi$ .

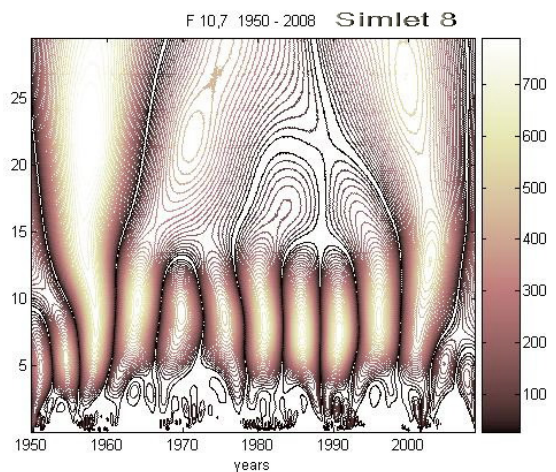


Figure 4b: The analysis of time series of  $F_{10.7}$  with the use of Simlet 8 mother wavelet.

In Figure 4a we show the Simlet 8 wavelet function. In Figure 4b we demonstrate the analysis of the radio emission  $F_{10.7}$  with the help of Simlet 8 mother wavelet. We see that the time-frequency parameters in this case have much more blurred contours around the maxima. Thus, the errors in determining the most probable values of the cycle's duration are increased compared with the study of a given series of observations using the wavelet Morlet.

**Meyer wavelet**

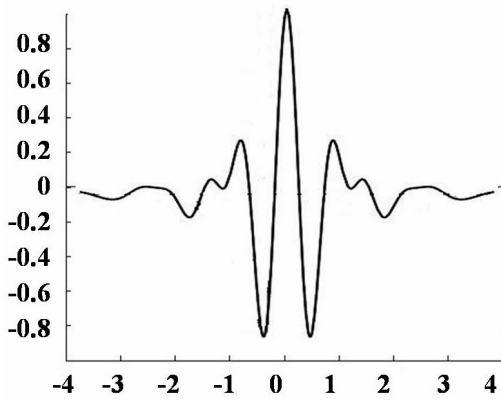


Figure 5a: The Meyer wavelet function  $\Psi$ .

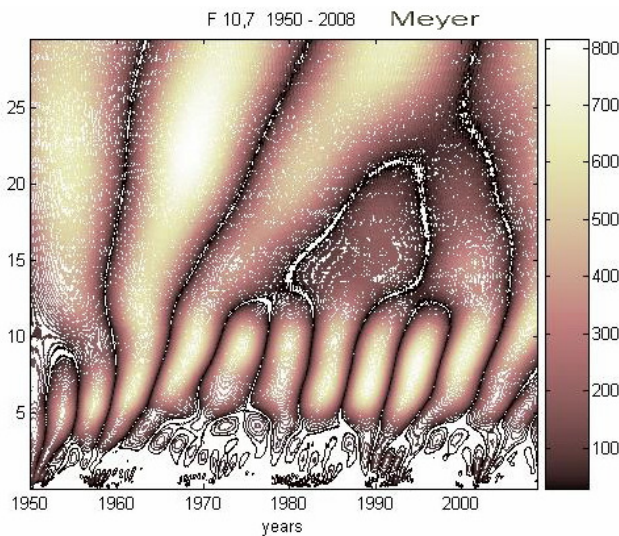


Figure 5b: The analysis of time series of  $F_{10.7}$  with the use of Meyer mother wavelet.

In Figure 5a we can see the Meyer wavelet function. Figure 5b demonstrates the analysis of the radio emission  $F_{10.7}$  with the help of Meyer mother wavelet. We also see that the time-frequency parameters in this case are not as good as in the case of Morlet wavelet. Thus, errors in the cycle's duration determination are increased compared with the study of the given series of observations using the wavelet Morlet.

**Gaussian wavelet**

The results of processing observation series using the Gaussian wavelet are very similar to the results of processing with the Morlet wavelet. Along with the basic 11-yr cycle of activity we also can see the quasi-biennial cyclicity.

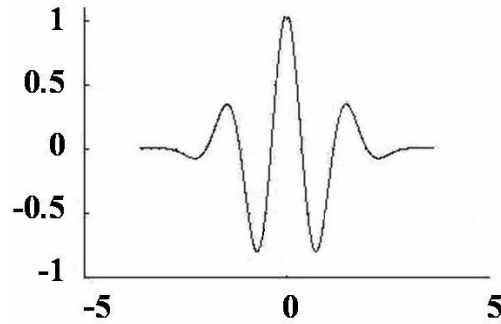


Figure 6a: The Gaussian 8 wavelet function  $\Psi$ .

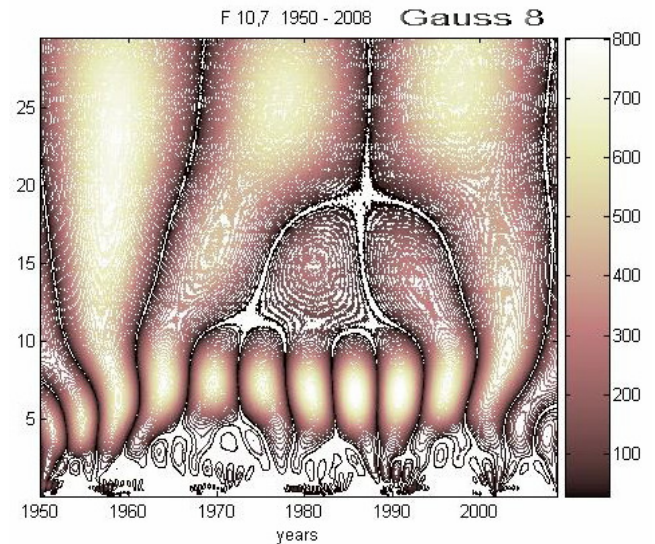


Figure 6b: The analysis of time series of  $F_{10.7}$  with the use of Gauss 8 mother wavelet.

In Figure 6a we show the Gauss 8 wavelet function. Figure 6b demonstrates the analysis of radio emission  $F_{10.7}$  with the help of Gaussian 8 mother wavelet. We see that the time-frequency parameters in this case are practically coinciding with the characteristics obtained with the use of the mother wavelet Morlet. Errors in determination of the most probable values of duration of the cycles are not more than in case when we use the Morlet wavelet. The differences between these wavelet studies are seen in small details, more concerning the quasi-biennial cycles.

We see that the cycles in this case have a shorter duration than the average value of 10.2 years. Perhaps this is due to the bad choice of the order of mother wavelet - Gauss 8 in our calculations. Perhaps, the Gauss wavelets of larger orders (10 -12) will give greater compliance with actual observations.

**Complex Morlet wavelet**

A complex Morlet wavelet is defined by:

$$\Psi(x) = \frac{1}{\sqrt{\pi f_b}} e^{2i\pi f_c x} e^{-\frac{x^2}{f_b}}$$

This wavelet depends on two parameters:  $f_b$  is a bandwidth parameter and  $f_c$  is a wavelet center frequency.

In Figure 7 we show the analysis of the time series of monthly averaged  $F_{10.7}$  (1950 – 2008 years) with the help of a complex Morlet 1-1.0 wavelet. In this case, there are two additional parameters that can be varied in accordance with the objectives set and, as in the case of Fourier analysis, we get the array of coefficients which give us information not only about the frequency-temporal distribution of the amplitude and about the frequency-temporal distribution of the phase of the signal. We can see that in this analysis the main cycle (11-year) is dominated and other cycles with lower amplitudes are suppressed. It is also seen that the duration of the 23<sup>rd</sup> cycle of activity is more than 12 years (this value we do not get with the use of real-valued wavelet Morlet as a result analysis).

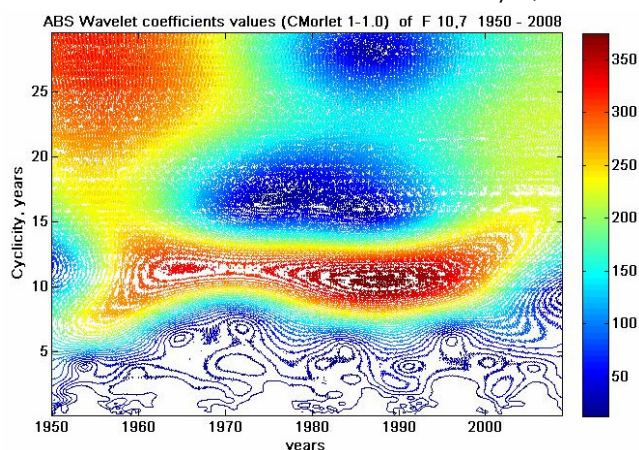


Figure 7: The time series of  $F_{10.7}$  (1950 - 2008) and analysis of the  $F_{10.7}$  data with the use of complex Morlet 1-1.0 wavelet.

### Summary and conclusion

Wavelet spectrum allows us not only to identify cycles, but also to analyze their change in time.

Each wavelet has its own characteristic features, so sometimes with the help of different wavelets the different properties of the analyzed signal can be better identified and highlighted.

In our work we analyze the main cyclicity of solar activity. The Morlet wavelet is the most suitable for this purpose. But we also see the existence of shorter cycles, with less intensity, which more successfully can be studied using wavelets suppressing the basic cycle and we can study the cyclicity of the second order in the intensity level.

With Morlet and Gauss real-valued and Morlet complex-valued wavelets we can study most accurately the solar cyclicity evolution in every moment of time. We can see also a number of periods, which, perhaps, are the harmonics of main period. The complex-valued Morlet and Gauss wavelet analyses give us the additional information about the signal phase evolution. Note that the results of our  $F_{10.7}$  study with the help of Simlet and Meyer wavelets are worse than using Morlet and Gauss wavelets. This results from the way in which Simlet and Meyer mother wavelets were constructed. During the solar cycles we see a clear maximum of the radiation flux  $F_{10.7}$  followed by its smooth decrease, similar to exponential decay. So we

confirm that for our observations of radio emission the most useful are mother wavelets which are constructed using convolution of Gaussian functions with the exponential function - Morlet and Gauss wavelets.

We can note that the Daubechies 10 wavelet-analysis (wider filter window) allows us to analyze the influence of the previous cycle on the next ones.

The analysis with all mother wavelets shows that the mean value of the 11-year cycle is about 10.2 years during the period 1950 - 2000. The complex-valued Morlet wavelet analysis shows the more long duration of 11-yr cyclicity for the cycle 23 – about 12 yr.

### Acknowledgments

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